Richardson Dees Primary School

September 2019

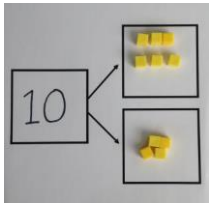
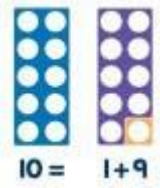
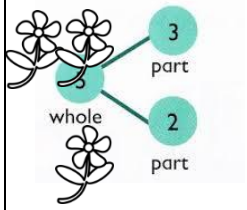
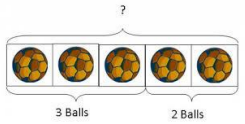
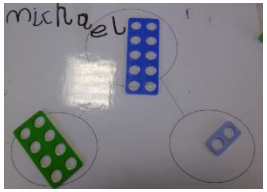
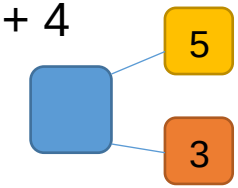
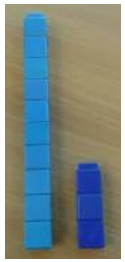
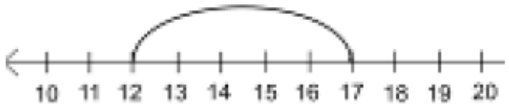
Calculation Policy:

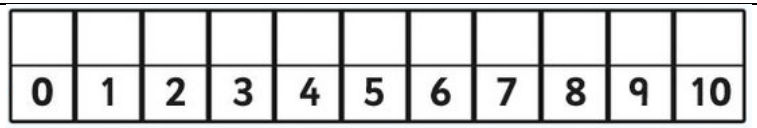
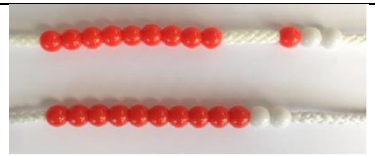
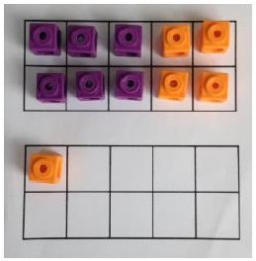
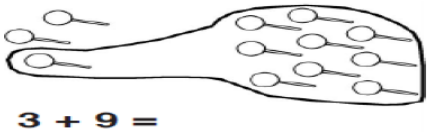
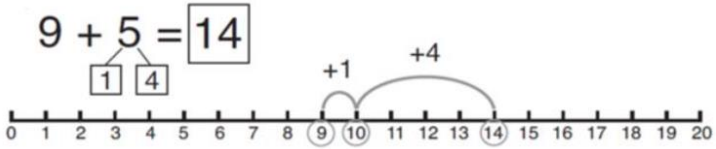
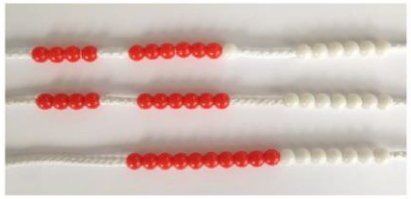
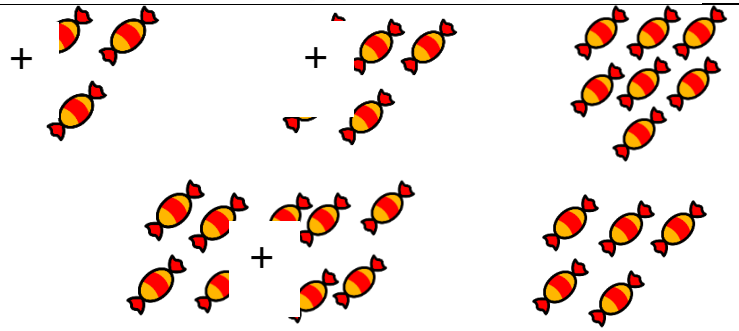
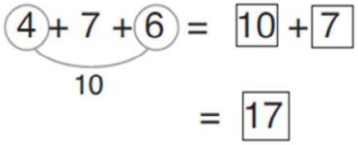
- Long term progression in calculations over the four operations: Addition, Subtraction, Multiplication and Division.

- Taken and adapted for Richardson Dees from The White Rose “Progression in Calculation” document.

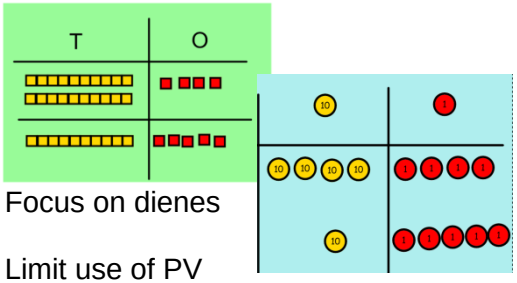
- HTU or HTO. We interchange between both ‘units’ and ‘ones’ so that children feel comfortable using both phrases.

Addition

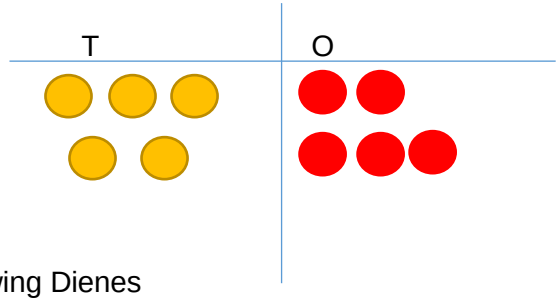
Objective and Strategies	Concrete	Pictorial	Abstract
Year 1 Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar.  	  Use pictures to add two numbers together as a group or in a bar.   	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Year 1 Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

	Anything that can be moved 1 by 1 such as counters and cubes.		
Year 1 Regrouping to make 10.	 6 + 5 = 11 Numicon Tens frame  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. Number track – to be used before number lines as seen below. This is an example of a pictorial representation which could be used. 	If I am at seven, how many more do I need to make 10. How many more do I add on now?
Year 2 Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Numicon Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

<table>
<tr>
<td><u>Year 2</u>

Column method- no regrouping</td>
<td>24 + 15=
Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.
 

Focus on dienes

Limit use of PV counters until late Year 3</td>
<td>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
 

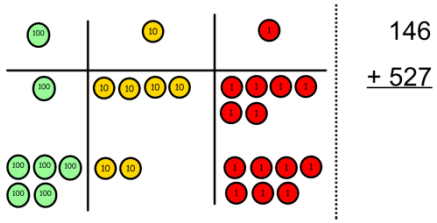
Drawing Dienes</td>
<td><u>Calculations</u>
21 + 42 =
21
+ 42</td>
</tr>
<tr>
<td><u>Year 3-6</u>

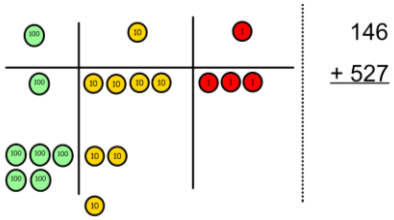
Column method- regrouping

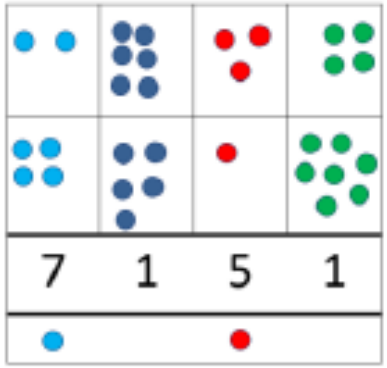
Y3 – up to 3 digits.

Y4 – up to 4 digits.

Y5 – more than 4 digits.

Y6 – Decimals with different amounts of numbers after the decimal point.</td>
<td>Make both numbers on a place value grid.
 

Add up the units and exchange 10 ones for one 10.
 

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</td>
<td>Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.
 

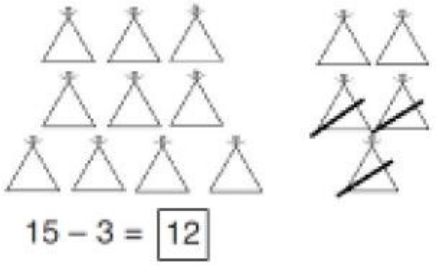
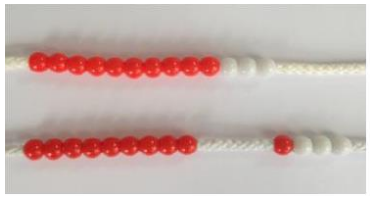
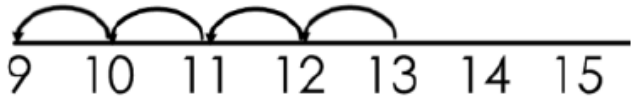
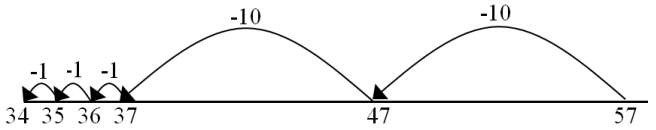
Place Value Counters.
Can also be done using Dienes. Drawing Dienes in Y3&4.</td>
<td>Expanded form to develop reasoning skills, especially when using increasingly larger numbers. Important to use in Year 3 to gain understanding and reasoning.

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.
This is expanded form:

20 + 5
40 + 8
60 + 13 = 73</td>
</tr>
</table>

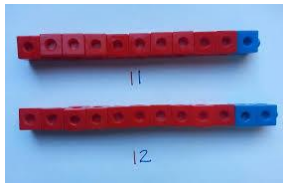
	This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\ 1 \end{array}$ As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. $\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ \hline 1\ 1 \end{array}$ $\begin{array}{rrrrrr} & £ & 2 & 3 & . & 5\ 9 \\ + & £ & & 7 & . & 5\ 5 \\ \hline & £ & 3 & 1 & . & 1\ 4 \\ & & 1 & 1 & & 1 \end{array}$ $\begin{array}{rrrrrr} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & \mathbf{0} \\ 5 & 9 & . & 7 & 7 & \mathbf{0} \\ + & 1 & . & 3 & \mathbf{0} & \mathbf{0} \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ \hline 2 & 1 & & 2 & & \end{array}$

Subtraction

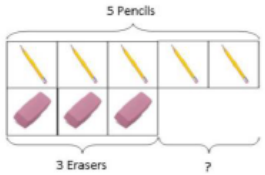
Objective and Strategies	Concrete	Pictorial	Abstract
Year 1 Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Year 1&2 + Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 - Use number track first before number line. Start at the bigger number and count back the smaller number showing the jumps on the number line. *(Needs to consistently be delivered across LKS2 as well)*  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

<table>
<tr>
<td><u>Year 1&2</u>

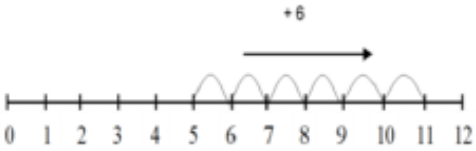
Find the difference</td>
<td>Compare amounts and objects to find the difference.

Use cubes to build towers or make bars to find the difference

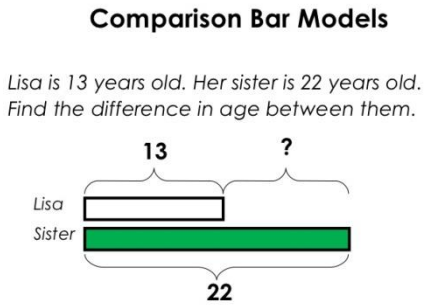
Use basic bar models with items to find the difference

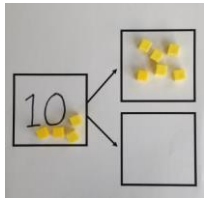
- Numicon – place numicon on top to visually see the difference</td>
<td> 

Count on to find the difference.

(Use concrete resources to understand why first).

Draw bars to find the difference between 2 numbers.

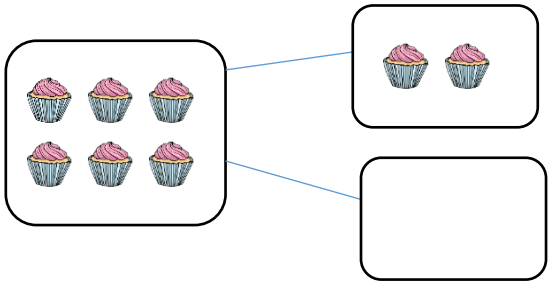
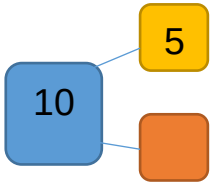
 
</td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</td>
</tr>
<tr>
<td><u>Year 1&2</u>

Part Part Whole Model</td>
<td> 

Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

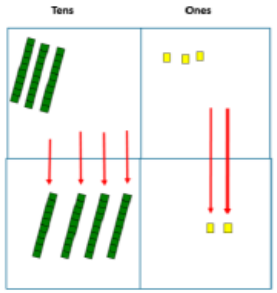
10 - 6 =
Cuisenaire to represent fact families.</td>
<td>Use a pictorial representation of objects to show the part part whole model.

  </td>
<td> 

Move to using numbers within the part whole model.</td>
</tr>
</table>

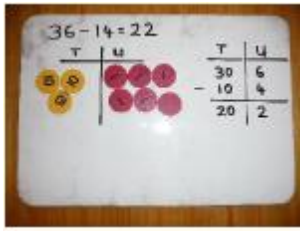
Year 2

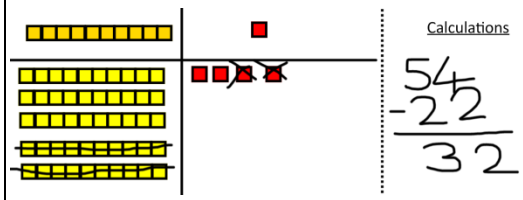
Column method without regrouping

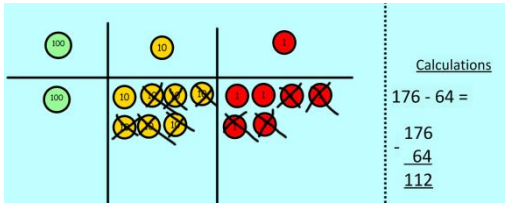


Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.







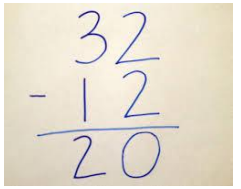
Draw the Base 10 or place value counters alongside the written calculation to help to show working.

- Use in UKS2 towards end of y4

47 - 24 = 23

40 + 7
- 20 + 4
20 + 3

This will lead to a clear written column subtraction.



Year 3-6

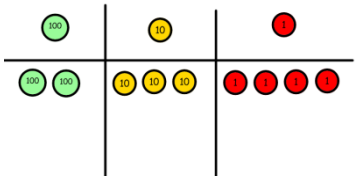
Column method with regrouping

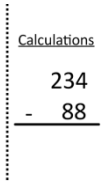
Y3 – up to 3 digits
Y4 – up to 4 digits
Y5 – more than 4 digits.
- Decimals

Y6 – decimals with various amounts of decimal places.

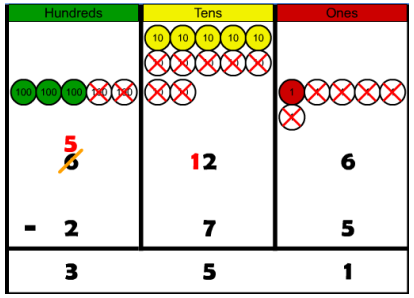
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

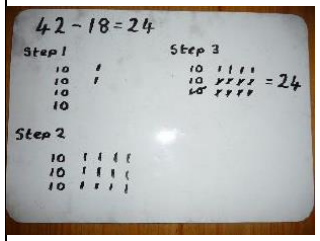




Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



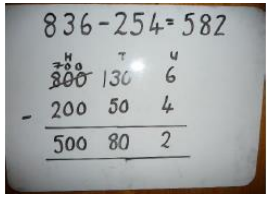
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



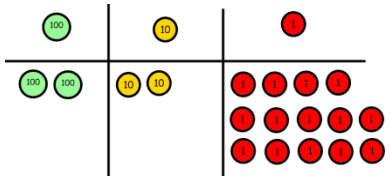
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

Expanded subtraction (below) to be done in Y3. Then use expanded and compact (second picture) method side by side.



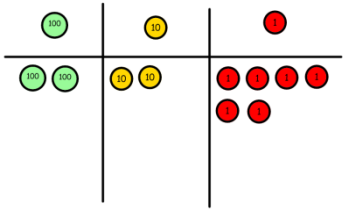
Children can start their formal written method by partitioning the number into clear place value columns.



Calculations

234
- 88

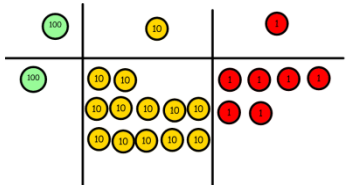
Now I can subtract my ones.



Calculations

234
- 88

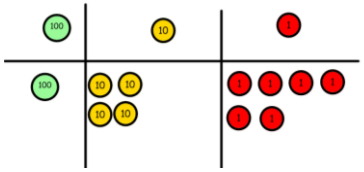
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Calculations

234
- 88

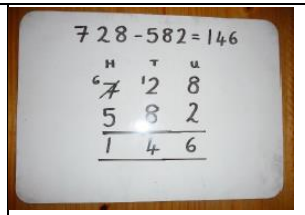
Now I can take away eight tens and complete my subtraction



Calculations

234
- 88
146

Show children how the concrete method links to the written method alongside your working. Cross out the



Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

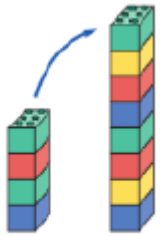
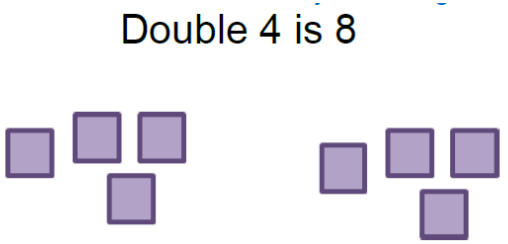
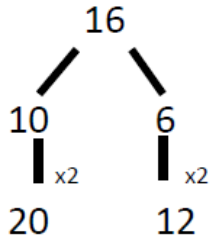
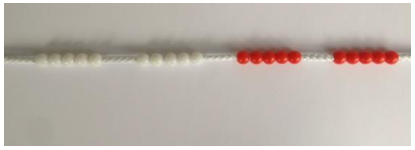
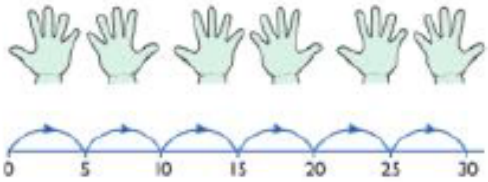
$$\begin{array}{r} \overset{5}{\cancel{2}}\ \overset{12}{\cancel{6}}\ \overset{1}{\cancel{3}}\ .\ \mathbf{0} \\ -\quad 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$

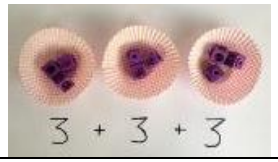
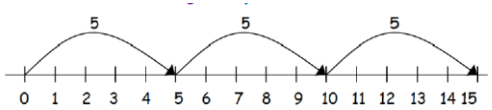
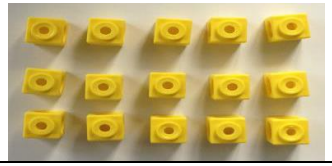
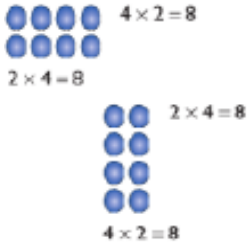
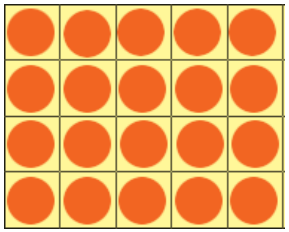
Y5 – same number of decimal places.

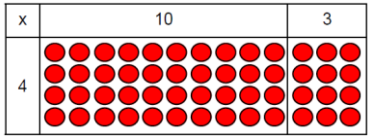
Y6 – wth different numbers of decimals places.

	numbers when exchanging and show where we write our new amount. Also use dienes in Y3 & Y4		

Multiplication

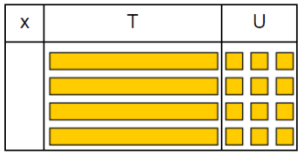
Objective and Strategies	Concrete	Pictorial	Abstract
Year 1&2 Doubling	Use practical activities to show how to double a number.  - numicon	Draw pictures to show how to double a number.  - numicon pictures	 Partition a number and then double each part before recombining it back together.
Year 1+ Counting in multiples	  Count in multiples supported by concrete objects in equal groups. - Numicon - Cuisenaire in Y2+	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Year 2 & 3 (some Y4) Repeated addition	  Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Year 2 & 3 (reinforce in y4) Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Year 3+ Grid Method	**13 x 4** Show the link with arrays to first introduce the grid method with counters	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.



4 rows of 10
4 rows of 3

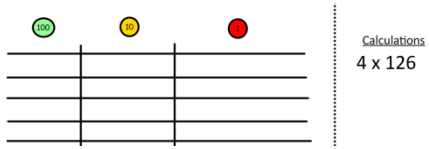
Move on to using Base 10 to move towards a more compact method (almost repeated addition).



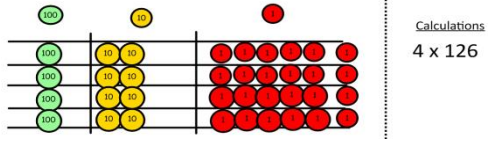
4 rows of 13

Use base 10 – then, once secure, move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.

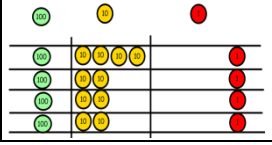
H T U



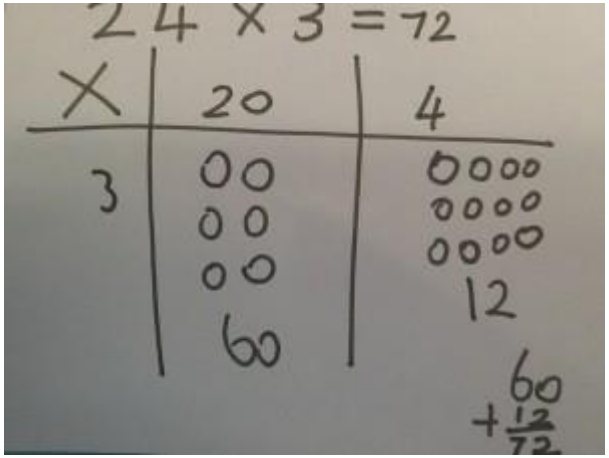
Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.



Also construct with PV counters before trying this.



35 x 7

×	30	5
7	210	35

210 + 35 = 245

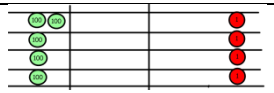
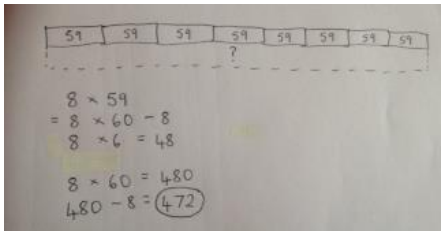
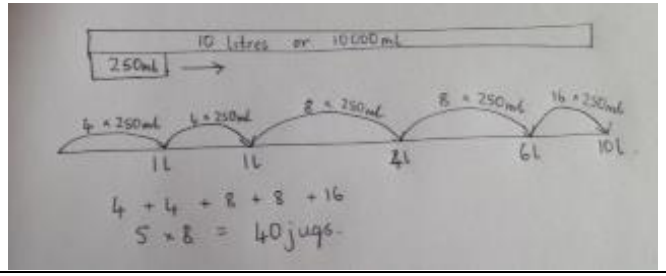
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

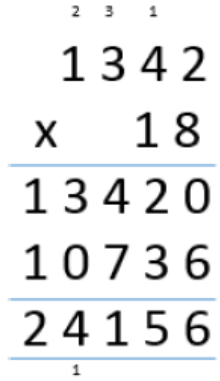
18 x 13

	10	8
10	100	80
3	30	24

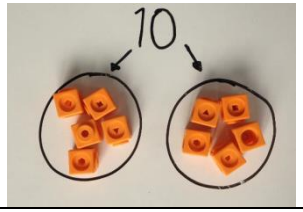
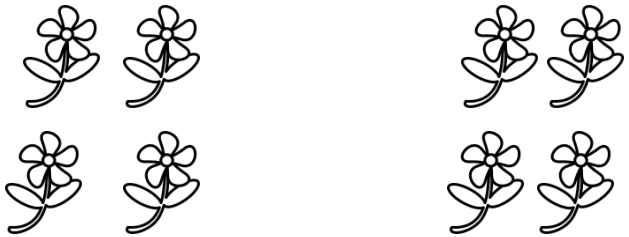
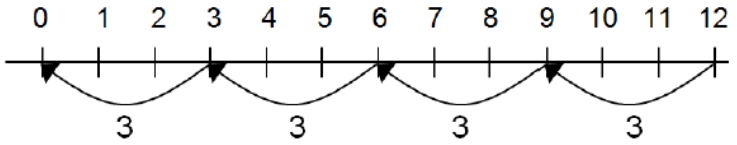
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

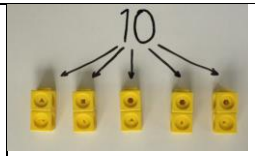
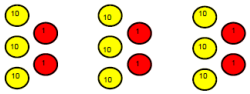
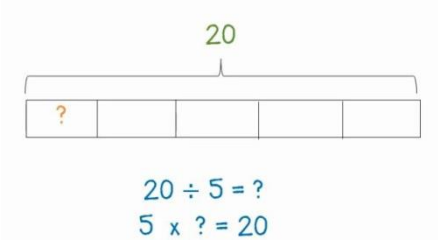
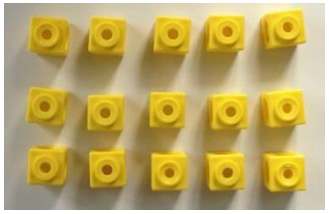
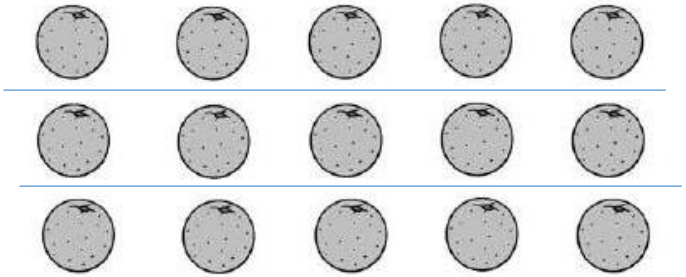
Once secure, move into column multiplication.

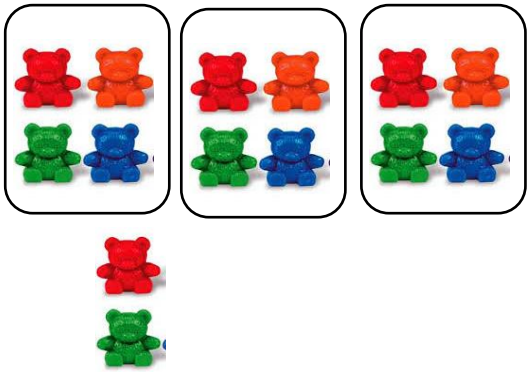
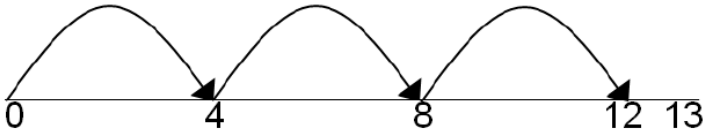
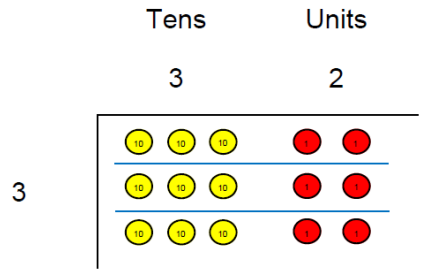
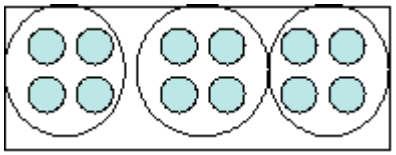
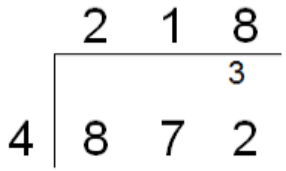
	 Then you have your answer.		
Year 4, 5, 6 Column multiplication Year 4 – two and three digit x 1 digit Year 5 – four numbers x 1 or 2 digit number Year 6 – 4 digits x 2 digits	Children can continue to be supported by place value counters at the stage of multiplication. Using place value counters and dienes (see grid method above for how to support understanding). Children must know the grid method first and use this as the concrete method before moving on. It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. Bar modelling can use cuisenaire.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods. (Children need to understand multiplication as repeated addition to use bar modelling for problem solving).   	Y5 & 6 - Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768

			This moves to the more compact   method.

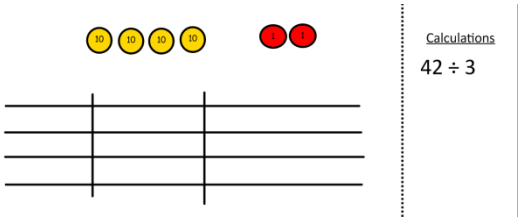
Division - In Y1 teach sharing and grouping alongside each other.

Objective and Strategies	Concrete	Pictorial	Abstract
Year 1 Sharing objects into groups 10 ÷ 2 as sharing	An understanding of what division is.  I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Year 1&2 Division as grouping 10 ÷ 2 as grouping (Developed over time as children progress up the school. Do this approach through times tables to develop an understanding in y1).	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

	 96 ÷ 3 = 32  	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	
Year 2, 3 & 4 Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences. Also use multiplication triangles	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 **“Fact Families”**

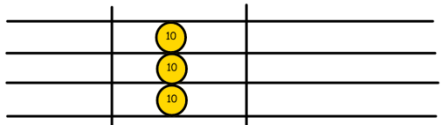
Year 3 & 4 Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over  This is 'sharing' model. Useful to see the relationship. Use arrays (as above) to support. It becomes obvious where the 2 spare are.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. **14 4**  This is the 'sharing' model. It is important children use both grouping and sharing models so they can see both ways. But, overall use arrays to show the remainder visually when the concrete is still needed.	Complete written divisions and show the remainder using r. 
Year 3 – 6 Y3 – Up to 2 digits by 1 digit Y4 – Up to 3 digits by 1 digit Y5 – Up to 4 digits by 1 digit (interpret remainders appropriately for context) Y6 – as above	96 ÷ 3  Use place value counters to divide using the bus stop method alongside	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Only for SEN children. Not practical for higher numbers. As soon as understood, move onto abstract. Encourage them to move towards counting in multiples to divide more efficiently. Can draw P.V. counters here if need the pictorial step.	Begin with divisions that divide equally with no remainder. 

Short division

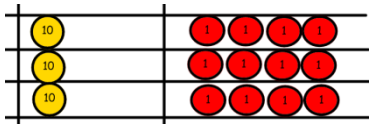


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.



To get the answer, we look how much in 1 group so the answer is 14.

Move onto divisions with a remainder.

```
      8  6   r  2
        3
   ┌──────────
5  │ 4  3  2
```

Finally move into decimal places to divide the total accurately.

```
         1  4  .  6
           16    21
   ┌──────────────
3  5 │ 5  1  1  .  0
```